ARIMA representation for GLIMPSE temperature

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1 Introduction

A random walk type changes in various averaged temperature series were studied several times (e.g. Gordon 1991, Kärner 1996). Simple ARIMA models (e.g. Box and Jenkins 1976, hereafter BJ76) represent a useful tool to examine the temporal variability.

2 Model choice and identification

Sample autocorrelations for the series $X(t)$, where t=1,2, ... n, and $x(t) = X(t+1) - X(t)$, up to the lag 20 years are shown in Fig. 1. There is a very slow decay in the autocorrelations for the GLIMPSE series. The correlations for the increment series have much simpler form, only the first of them (i.e for the lag k=1 year) is significantly non-zero. Autocorrelation for the surface air temperature anomaly series from Stockholm (calculated from daily series, Klein Tank et al. 2002) is shown on right panel for comparison.

Such behavior of autocorrelations advises using MA(1) model to represent the increment series:

$$x(t) = \Theta_0 + (1 - \Theta_1 B)a(t), \tag{1}$$

where B is a backward shift operator, i.e $Ba(t) = a(t-1)$, $a(t)$ is white noise. Θ_0 describes average tendency in the sample and it is calculated from the expression $\Theta_0 = (1/(n-1)) \sum_{i=1}^{n-1} x(i)$

Standard error for the increments mean value $\overline{x}$ (BJ76):

$$\sigma_{\overline{x}} = \sqrt{\frac{C(0)(1+2r(1))}{n-1}}, \tag{2}$$

can be used to estimate whether Θ_0 should be nonzero in the model (1). Here $C(0)$ and $r(1)$ stand for the variance and lag one autocorrelation for $x(t)$.

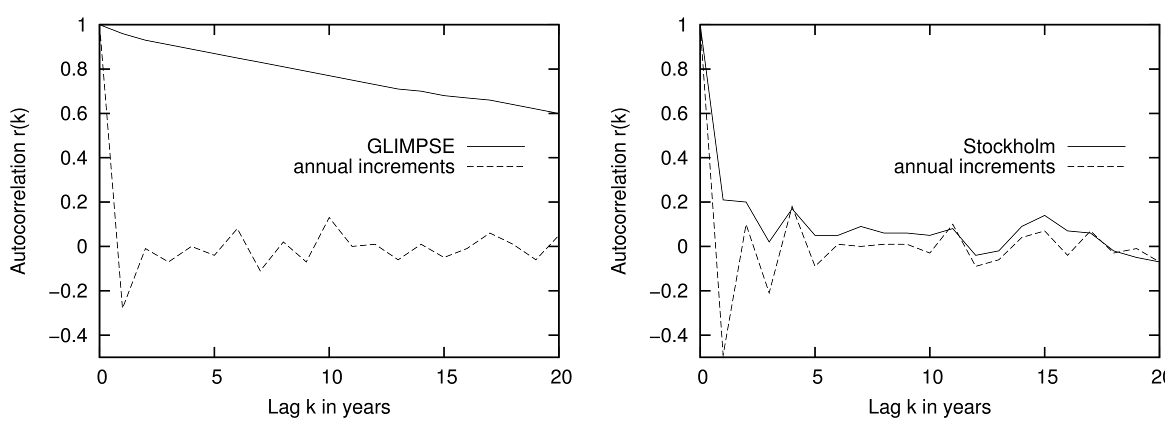

Figure 1: Autocorrelations for 2 series and their annual increments: GLIMPSE 1600-1999 (left); Stockholm 1756-2996 (right)

Table 1: Fitted parameter values

Variable	Time	n	Θ_1	Θ_0	error	Q
GLIMPSE	1600-1999	400	0.341	0.0035	0.0023	29.9
Stockholm	1756-2005	250	0.878	0.0013	0.007	29.8
Global MSU, monthly	(Kärner	2002)				
T_{TROP}	1979-2001	264	0.354	-	-	22.4
T_{STRAT}	1979-2001	264	-0.50	-	-	28.3

3 Results

Fitted values for the parameters are shown in Table **??**. Temporal variability for some other temperature series can be also described by means of the ARIMA (0,1,1) model. The second row shows the results for the Stockholm series (1756-2005). The third and fourth rows show results for the MSU based global tropospheric and stratospheric temperature anomaly series, respectively (Kärner 2002). The portmanteau statistic Q is calculated from 24 autocorrelations. Its critical value at 95% level is 35.2, showing a satisfactory representation for all 4 series.

4 Conclusions

One and the same model type (0,1,1) appears to be applicable for various temperature series. The model expresses temperature changes by means of random impulses. BJ76 interpretes the process as random walk in a noisy environment. The trend term Θ_0 appears to be important in case of annual series.

References

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