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Comment on “A Semi-Empirical Approach to Projecting Future Sea-Level Rise” by S. Rahmstorf

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**Abstract**

We revisit the application of conventional statistical methods, assuming stationarity, to the problem of projected sea-level rise. Trends and autocorrelations in the data give optimistic estimates of the correlation coefficient and its significance level. Estimation of the regression coefficient is not robust and this questions the statistical model put forward. We suggest the application of methods commonly used within econometrics.

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Rahmstorf (ref. 1), henceforth R07, convincingly argues for the use of semi-empirical models for estimating sea level response to future warming of the climate system. He hypothesize that the rate of global sea level change is expected to be proportional to the global surface temperature departure from its equilibrium value. This hypothesis is then statistically tested on observational data and a correlation coefficient of 0.88 with the associated p-value of $1.6 \times 10^{-8}$ is reported, while the regression slope is reported as 3.4 mm/year per deg. C.

The statistical analysis of Rahmstorf is, however, based on a flawed application of statistics, in that the trend in both series is evident and therefore basic assumptions of the statistical methods used are violated. This could give seriously misleading conclusions about inference (ref. 2) due to spurious or ‘nonsense’ correlation (ref. 3) and therefore the projected range of future sea level rise is questionable.

In order to illustrate this, we redid the analysis of R07, with methodological details as follows: As in R07, we used annually averaged global mean temperature and sea level. Nonlinear trends of both series were determined as the first reconstructed component in an SSA-analysis with an embedding dimension of 15 years. Prior to the SSA-analysis, both series were extended forwards and backwards by linear extrapolation based on the nearest 15 years of data. The nonlinear trend of mean sea level was subsequently differentiated by in each point calculating the slope of a ±5 year least squares fit to obtain a ‘rate of sea level change’ series.

Our critique of R07 now goes as follows: First, the correlation coefficient $\rho$ between two time series $x_t$ and $y_t$ with deterministic time trends is defined as

$$\rho = \frac{E\{(x_t - E(x_t))(y_t - E(y_t))\}}{\sqrt{E\{(x_t - E(x_t))^2\}E\{(y_t - E(y_t))^2\}}},$$

where $E(\bullet)$ denotes expectation value. The correlation coefficient thus measures to which degree there are coincident departures of the two time series from their respective expectation values. When estimating the correlation coefficient between filtered versions of temperature and rate of sea level change, R07 substitutes the expectation values by the sample average. This assumes stationarity of the series (ref. 4), which is obviously violated by the two series. As an illustration, when redoing the analysis of R07, we approximated the expectation values of the two series by the much more realistic choice of a linear trend. Doing that, the estimated correlation coefficient drops to 0.68. Also the corresponding regression coefficient increases from 3.3 mm/year per deg. C to 5.8 mm/year per deg. C. This non-robust result underlines that the issue of correct statistical modelling is not an academic one and questions the model put forward in R07.

Secondly, there is the point of establishing the significance of the correlation coefficient found, i.e. 'how likely is it to get the result by pure chance'. Rahmstorf appears to have estimated the p-value of the correlation coefficient -1.6x10$^{-8}$ - using a Student's t distribution assuming 24 degrees of freedom. The number of degrees of freedom apparently comes from assuming that the 24 bins of 5 year length are statistically independent. However, both data series were low-pass filtered in R07 'by computing non-linear trend lines with embedding period of 15 years'. Due to the autocorrelation introduced by the averaging procedure neighbouring 5-year bins can not be assumed to be statistically independent. A better approximation would be to put the number of degrees of freedom equal to the effective sample size calculated as 120/15 = 8 (ref 4). Using our value of 0.68 for the correlation coefficient, we get a corresponding p-value of 0.97.

Finally, R07 used the t-test for making inferences about the correlation coefficient. This is based on the assumption of independent and identically distributed (i.i.d.) data, and the data analysed in R07 are -

due to the trend - not i.i.d. This reservation also also goes for the estimated confidence interval and therefore the range of projected sea level changes by year 2100.

A thorough analysis of the problem would include the application of difference stationary time series analysis methods (ref 5). There is a rich tradition of such analysis within the field of econometrics, which with advantage could be transferred to the problem analysed here and to other problems within climate science. European Science Foundation has realised the importance of this by supporting an ‘exploratory workshop’ with the title ‘Econometric time-series analysis applied to climate research’, held on 6-7 September 2007 in Frascati, Italy with 30 participants from econometrics and climate science.

**References**


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